NAG Toolbox for MATLAB

f08hb

1 Purpose

f08hb computes selected eigenvalues and, optionally, eigenvectors of a real n by n symmetric band matrix A of bandwidth $(2k_d + 1)$. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

2 Syntax

```
[ab, q, m, w, z, jfail, info] = f08hb(jobz, range, uplo, kd, ab, vl, vu,
il, iu, abstol, 'n', n)
```

3 Description

The symmetric band matrix A is first reduced to tridiagonal form, using orthogonal similarity transformations. The required eigenvalues and eigenvectors are then computed from the tridiagonal matrix; the method used depends upon whether all, or selected, eigenvalues and eigenvectors are required.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Demmel J W and Kahan W 1990 Accurate singular values of bidiagonal matrices *SIAM J. Sci. Statist. Comput.* **11** 873–912

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz – string**

If **jobz** = 'N', compute eigenvalues only.

If **jobz** = 'V', compute eigenvalues and eigenvectors.

Constraint: **jobz** = 'N' or 'V'.

2: **range – string**

If **range** = 'A', all eigenvalues will be found.

If **range** = 'V', all eigenvalues in the half-open interval (**vl**, **vu**] will be found.

If **range** = 'I', the **il**th to **iu**th eigenvalues will be found.

Constraint: **range** = 'A', 'V' or 'I'.

3: **uplo – string**

If **uplo** = 'U', the upper triangular part of A is stored.

If **uplo** = 'L', the lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

4: **kd – int32 scalar**

If **uplo** = 'U', the number of superdiagonals, k_d, of the matrix A.

If **uplo** = 'L', the number of subdiagonals, k_d, of the matrix A.

Constraint: **kd** ≥ 0.

5: **ab**(**ldab**,$*$) **– double array**

The first dimension of the array **ab** must be at least **kd** $+ 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of the n by n symmetric band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_d + 1 + i - j, j)$ for $\max(1j - k_d) \leq i \leq j$;
if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j, j)$ for $j \leq i \leq \min(nj + k_d)$.

6: **vl – double scalar**
7: **vu – double scalar**

If **range** = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.

If **range** = 'A' or 'I', **vl** and **vu** are not referenced.

Constraint: if **range** = 'V', **vl** $<$ **vu**.

8: **il – int32 scalar**
9: **iu – int32 scalar**

If **range** = 'I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned.

If **range** = 'A' or 'V', **il** and **iu** are not referenced.

Constraints:

if $\mathbf{n} = 0$, $\mathbf{il} = 1$ and $\mathbf{iu} = 0$;
if $\mathbf{n} > 0$, $1 \leq \mathbf{il} \leq \mathbf{iu} \leq \mathbf{n}$.

10: **abstol – double scalar**

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a, b]$ of width less than or equal to

$$\mathbf{abstol} + \epsilon \max(|a|, |b|),$$

where ϵ is the ***machine precision***. If **abstol** is less than or equal to zero, then $\epsilon\|T\|_1$ will be used in its place, where T is the tridiagonal matrix obtained by reducing A to tridiagonal form. Eigenvalues will be computed most accurately when **abstol** is set to twice the underflow threshold $2 \times$ x02am( ), not zero. If this function returns with **info** > 0, indicating that some eigenvectors did not converge, try setting **abstol** to $2 \times$ x02am( ). See Demmel and Kahan 1990.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldq, ldz, work, iwork

5.4 Output Parameters

1: **ab(ldab,$*$) – double array**

The first dimension of the array **ab** must be at least $\mathbf{kd} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

ab contains values generated during the reduction to tridiagonal form.

The first superdiagonal and the diagonal of the tridiagonal matrix T are returned in **ab** using the same storage format as described above.

2: **q(ldq,$*$) – double array**

The first dimension, **ldq**, of the array **q** must satisfy

if $\mathbf{jobz} = \text{'V'}$, $\mathbf{ldq} \geq \max(1, \mathbf{n})$;
$\mathbf{ldq} \geq 1$ otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $\mathbf{jobz} = \text{'V'}$, the n by n orthogonal matrix used in the reduction to tridiagonal form.

If $\mathbf{jobz} = \text{'N'}$, the array **q** is not referenced.

3: **m – int32 scalar**

The total number of eigenvalues found.

If $\mathbf{range} = \text{'A'}$, $\mathbf{m} = \mathbf{n}$.

If $\mathbf{range} = \text{'V'}$, the exact value of **m** is not known in advance, but will satisfy $0 \leq \mathbf{m} \leq \mathbf{n}$.

If $\mathbf{range} = \text{'I'}$, $\mathbf{m} = \mathbf{iu} - \mathbf{il} + 1$.

4: **w($*$) – double array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

The first **m** elements contain the selected eigenvalues in ascending order.

5: **z(ldz,$*$) – double array**

The first dimension, **ldz**, of the array **z** must satisfy

if $\mathbf{jobz} = \text{'V'}$, $\mathbf{ldz} \geq \max(1, \mathbf{n})$;
$\mathbf{ldz} \geq 1$ otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{m})$ if $\mathbf{jobz} = \text{'V'}$, and at least 1 otherwise

If $\mathbf{jobz} = \text{'V'}$, then if $\mathbf{info} = 0$, the first m columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the ith column of Z holding the eigenvector associated with $\mathbf{w}(i)$.

If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in **jfail**.

If $\mathbf{jobz} = \text{'E'}$, **z** is not referenced.

Note: you must ensure that at least $\max(1, \mathbf{m})$ columns are supplied in the array **z**; if $\mathbf{range} = \text{'V'}$, the exact value of **m** is not known in advance and an upper bound must be used.

6: **jfail**($*$) – **int32 array**

Note: the dimension of the array **jfail** must be at least $\max(1, \mathbf{n})$.

If **jobz** = 'V', then if **info** $= 0$, the first **m** elements of **jfail** are zero.

If **info** > 0, **jfail** contains the indices of the eigenvectors that failed to converge.

If **jobz** = 'E', **jfail** is not referenced.

7: **info** – **int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **range**, 3: **uplo**, 4: **n**, 5: **kd**, 6: **ab**, 7: **ldab**, 8: **q**, 9: **ldq**, 10: **vl**, 11: **vu**, 12: **il**, 13: **iu**, 14: **abstol**, 15: **m**, 16: **w**, 17: **z**, 18: **ldz**, 19: **work**, 20: **iwork**, 21: **jfail**, 22: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, then i eigenvectors failed to converge. Their indices are stored in array **jfail**. Please see **abstol**.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is proportional to $k_d n^2$ if **jobz** = 'N', and is proportional to n^3 if **jobz** = 'V' and **range** = 'A', otherwise the number of floating-point operations will depend upon the number of computed eigenvectors.

The complex analogue of this function is f08hp.

9 Example

```
jobz = 'Vectors';
range = 'Values in range';
uplo = 'U';
kd = int32(2);
ab = [0, 0, 3, 4, 5;
      0, 2, 3, 4, 5;
      1, 2, 3, 4, 5];
vl = -3;
vu = 3;
il = int32(0);
iu = int32(0);
```

```
abstol = 0;
[abOut, q, m, w, z, jfail, info] = ...
    f08hb(jobz, range, uplo, kd, ab, vl, vu, il, iu, abstol)
```

```
abOut =
         0         0    3.0000    6.9338    1.5841
         0    3.6056    6.9682   -2.3328   -0.2640
    1.0000    5.4615    8.9115    2.8591   -3.2322
q =
    1.0000         0         0         0         0
         0    0.5547    0.0827    0.6078    0.5622
         0    0.8321   -0.0551   -0.4052   -0.3748
         0         0    0.7960    0.3491   -0.4944
         0         0    0.5970   -0.5870    0.5468
m =
           2
w =
   -2.6633
    1.7511
         0
         0
         0
z =
   -0.6238   -0.5635
    0.2575    0.3896
    0.5900   -0.4008
   -0.4308    0.5581
   -0.1039   -0.2421
jfail =
           0
           0
           0
           0
           0
info =
           0
```